Exercise 27

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*Derive (D) in system **T**.*

The schema (that is, all instances thereof) $\Box A \supset A$ is in **T** by axiom (T). Also by axiom (T), the schema $\Box\neg A \supset \neg A$ is in **T**, and (by an instance of the CL-tautology $(P \supset Q) \supset (\neg Q \supset \neg P)$ and *modus ponens*) hence all instances of $\neg\neg A \supset \neg\Box\neg A$ are also in **T**. Making use of (CL-)equivalence of $\neg\neg A$ and A, and applying the definition of $\Diamond$, this means that all instances of the schema $A \supset \Diamond A$ are also in **T**. Again by *modus ponens* of $\Box A \supset A$ and the appropriate instance of the CL-tautology $A \supset (B \supset (A \wedge B))$, we can derive $(A \supset \Diamond A) \supset ((\Box A \supset A) \wedge (A \supset \Diamond A))$, from which we can obtain $(\Box A \supset A) \wedge (A \supset \Diamond A)$, once again by *modus ponens*. One final application of *modus ponens* with an instance of the CL-tautology $((P \supset Q) \wedge (Q \supset R)) \supset (P \supset R)$ suffices to derive $\Box A \supset \Diamond A$.