The following aliases were used:
$B \equiv ((\mathrm{y} > 1) \land (\mathrm{x} = 3))$
$Q \equiv ((\mathrm{x} + \mathrm{y}) < 19)$

$$\dfrac{\dfrac{\dfrac{\overset{(1)}{((B \land Q) \supset ((\mathrm{x} > \mathrm{y})[^{(\mathrm{x}+19)}_{\mathrm{x}}]))}}{(B \land Q)\ \{\mathrm{x} \leftarrow (\mathrm{x}+19)\}\ (\mathrm{x} > \mathrm{y})}\ \text{(H1)} \quad \dfrac{\overset{(2)}{((B \land \neg Q) \supset ((\mathrm{x} > \mathrm{y})[^{(\mathrm{y}+\mathrm{y})}_{\mathrm{x}}]))}}{(B \land \neg Q)\ \{\mathrm{x} \leftarrow (\mathrm{y}+\mathrm{y})\}\ (\mathrm{x} > \mathrm{y})}\ \text{(H1)}}{B\ \{\text{if } Q \text{ then } \mathrm{x} \leftarrow (\mathrm{x}+19) \text{ else } \mathrm{x} \leftarrow (\mathrm{y}+\mathrm{y})\}\ (\mathrm{x} > \mathrm{y})}\ \text{(H3)}}{(\mathrm{y} > 1)\ \{\text{begin } \mathrm{x} \leftarrow 3;\ \text{if } Q \text{ then } \mathrm{x} \leftarrow (\mathrm{x}+19) \text{ else } \mathrm{x} \leftarrow (\mathrm{y}+\mathrm{y}) \text{ end}\}\ (\mathrm{x} > \mathrm{y})}\ \text{(T1)}$$

The following aliases were used:
$P \equiv ((\mathrm{x} = 3) \land (\mathrm{y} = 1))$
$Q \equiv (\mathrm{y} = 10)$

$$\dfrac{\overset{(1)}{(P \supset \mathrm{I1})} \quad \dfrac{\dfrac{\overset{(2)}{((\mathrm{I1} \land (\mathrm{x} > 0)) \supset ((\mathrm{I1}[^{(\mathrm{x} \dot{-} 1)}_{\mathrm{x}}])[^{(\mathrm{y}+3)}_{\mathrm{y}}]))}}{(\mathrm{I1} \land (\mathrm{x} > 0))\ \{\mathrm{y} \leftarrow (\mathrm{y}+3)\}\ (\mathrm{I1}[^{(\mathrm{x} \dot{-} 1)}_{\mathrm{x}}])}\ \text{(H1)}}{(\mathrm{I1} \land (\mathrm{x} > 0))\ \{\text{begin } \mathrm{y} \leftarrow (\mathrm{y}+3);\ \mathrm{x} \leftarrow (\mathrm{x} \dot{-} 1) \text{ end}\}\ \mathrm{I1}}\ \text{(T2)} \quad \overset{(3)}{((\mathrm{I1} \land \neg(\mathrm{x} > 0)) \supset Q)}}{P\ \{\text{while } (\mathrm{x} > 0) \text{ do begin } \mathrm{y} \leftarrow (\mathrm{y}+3);\ \mathrm{x} \leftarrow (\mathrm{x} \dot{-} 1) \text{ end}\}\ Q}\ \text{(H4)}$$

Invariant $I1$: $(((3 * \mathrm{x}) + (1 * \mathrm{y})) = \mathrm{c1})$
Choose $c1$ so that $(P \supset (((3 * \mathrm{x}) + (1 * \mathrm{y})) = \mathrm{c1}))$ is valid in $\mathbb{N}$.

The following aliases were used:
$P \equiv (\mathrm{divisor} > 0)$
$Q \equiv ((\mathrm{dividend} = (\mathrm{rem} + (\mathrm{quot} * \mathrm{divisor}))) \land (\mathrm{rem} < \mathrm{divisor}))$
$I \equiv \mathrm{I1}$
$B \equiv (\mathrm{rem} \geq \mathrm{divisor})$

$$\dfrac{\dfrac{\overset{(1)}{((P \wedge (\text{rem} = \text{dividend})) \supset (I[^{0}_{\text{quot}}]))}}{(P \wedge (\text{rem} = \text{dividend}))\ \{\text{quot} \leftarrow 0\}\ I}\ (\text{H1})}{P\ \{\text{begin rem} \leftarrow \text{dividend};\ \text{quot} \leftarrow 0\ \text{end}\}\ I}\ (\text{T1}) \quad \dfrac{\dfrac{\overset{(2)}{((I \wedge B) \supset ((I[^{(\text{quot}+1)}_{\text{quot}}])[^{(\text{rem} \dot{-} \text{divisor})}_{\text{rem}}]))}}{(I \wedge B)\ \{\text{rem} \leftarrow (\text{rem} \dot{-} \text{divisor})\}\ (I[^{(\text{quot}+1)}_{\text{quot}}]}}{(I \wedge B)\ \{\text{begin rem} \leftarrow (\text{rem} \dot{-} \text{divisor});\ \text{quot} \leftarrow (\text{quot} +}$$

$$P\ \{\text{begin begin rem} \leftarrow \text{dividend};\ \text{quot} \leftarrow 0\ \text{end};\ \text{while}\ B\ \text{do begin rem} \leftarrow (\text{rem} \dot{-} \text{divisor});\ \text{quot} \leftarrow$$

Invariant $I1$: $(((1 * \text{rem}) + (\text{divisor} * \text{quot})) = \text{c1})$
Choose $c1$ so that $(P \supset (((1 * \text{rem}) + (\text{divisor} * \text{quot})) = \text{c1}))$ is valid in $\mathbb{N}$.

$$\dfrac{\dfrac{\overset{(1)}{(((\text{x} > 0) \wedge ((\text{x}+\text{y}) = \text{z})) \supset ((((\text{x}+\text{y}) = \text{z})[^{(\text{y}+1)}_{\text{y}}])[^{(\text{x} \dot{-} 1)}_{\text{x}}]))}}{((\text{x} > 0) \wedge ((\text{x}+\text{y}) = \text{z}))\ \{\text{x} \leftarrow (\text{x} \dot{-} 1)\}\ (((\text{x}+\text{y}) = \text{z})[^{(\text{y}+1)}_{\text{y}}])}\ (\text{H1})}{((\text{x} > 0) \wedge ((\text{x}+\text{y}) = \text{z}))\ \{\text{begin x} \leftarrow (\text{x} \dot{-} 1);\ \text{y} \leftarrow (\text{y}+1)\ \text{end}\}\ ((\text{x}+\text{y}) = \text{z})}\ (\text{T2})$$

$$\dfrac{\dfrac{\overset{(1)}{(((\text{y} > 0) \wedge (\text{x} > \text{y})) \supset ((\text{z} > \text{y})[^{\text{x}}_{\text{z}}]))}}{((\text{y} > 0) \wedge (\text{x} > \text{y}))\ \{\text{z} \leftarrow \text{x}\}\ (\text{z} > \text{y})}\ (\text{H1}) \quad \dfrac{\dfrac{\overset{(2)}{((((\text{y} > 0) \wedge \neg(\text{x} > \text{y})) \wedge (\text{z} = \text{y})) \supset ((\text{z} > \text{y})[^{(\text{y} \dot{-} 1)}_{\text{y}}]))}}{(((\text{y} > 0) \wedge \neg(\text{x} > \text{y})) \wedge (\text{z} = \text{y}))\ \{\text{y} \leftarrow (\text{y} \dot{-} 1)\}\ (\text{z} > \text{y})}\ (\text{H1}}{((\text{y} > 0) \wedge \neg(\text{x} > \text{y}))\ \{\text{begin z} \leftarrow \text{y};\ \text{y} \leftarrow (\text{y} \dot{-} 1)\ \text{end}\}\ (\text{z} > \text{y})}\ (\text{T}}{(\text{y} > 0)\ \{\text{if}\ (\text{x} > \text{y})\ \text{then z} \leftarrow \text{x}\ \text{else begin z} \leftarrow \text{y};\ \text{y} \leftarrow (\text{y} \dot{-} 1)\ \text{end}\}\ (\text{z} > \text{y})}\ (\text{I}$$

$$
\cfrac{\overset{(1)}{(((x = 2) \wedge (y = 3)) \supset I1)} \qquad \cfrac{\cfrac{\overset{(2)}{((I1 \wedge (x \neq y)) \supset ((I1[{}^{(y+2)}_{y}])[{}^{(x+1)}_{x}]))}}{(I1 \wedge (x \neq y))\ \{x \leftarrow (x + 1)\}\ (I1[{}^{(y+2)}_{y}])}\ (H1)}{(I1 \wedge (x \neq y))\ \{\text{begin } x \leftarrow (x + 1);\ y \leftarrow (y + 2) \text{ end}\}\ I1}\ (T2) \qquad \overset{(3)}{((I1 \wedge \neg(x \neq y))}}{((x = 2) \wedge (y = 3))\ \{\text{while } (x \neq y) \text{ do begin } x \leftarrow (x + 1);\ y \leftarrow (y + 2) \text{ end}\}\ (x = 1)}
$$

Invariant $I1$ is either: $((1 * y) = ((2 * x) + c1))$ or: $(((1 * y) + c1) = (2 * x))$; $c1$ should be positive, and $(((x = 2) \wedge (y = 3)) \supset ((1 * y) = ((2 * x) + c1)))$ or $(((x = 2) \wedge (y = 3)) \supset (((1 * y) + c1) = (2 * x)))$ should hold, respectively.

$$
\cfrac{\cfrac{\overset{(1)}{(((y = 5) \wedge (v = 3)) \supset (((a = 7)[{}^{7}_{a}])[{}^{(5+y)}_{v}]))}}{((y = 5) \wedge (v = 3))\ \{v \leftarrow (5 + y)\}\ ((a = 7)[{}^{7}_{a}])}\ (H1)}{((y = 5) \wedge (v = 3))\ \{\text{begin } v \leftarrow (5 + y);\ a \leftarrow 7 \text{ end}\}\ (a = 7)}\ (T2)
$$